Alma Mater Studiorum Università di Bologna
Laurea Magistrale in Fisica del Sistema Terra
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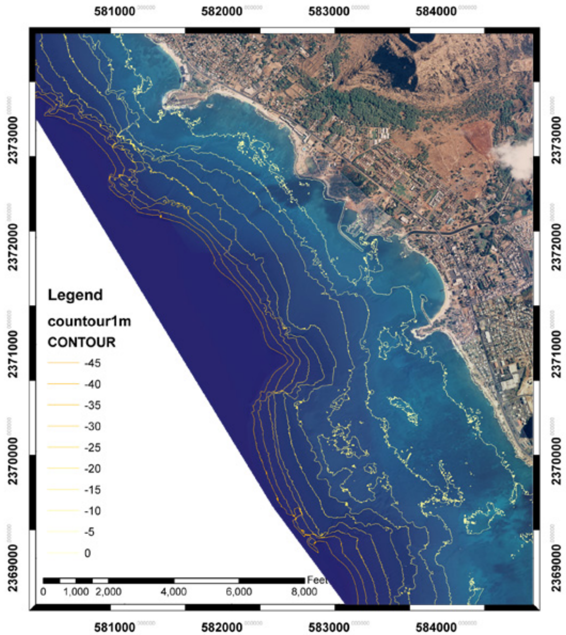
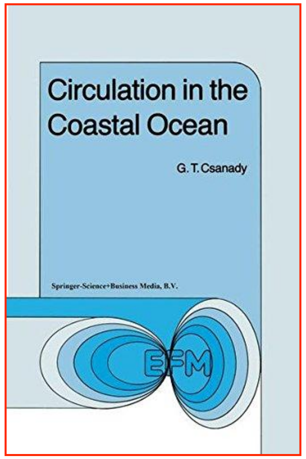
# The role of bathymetry

# Part1



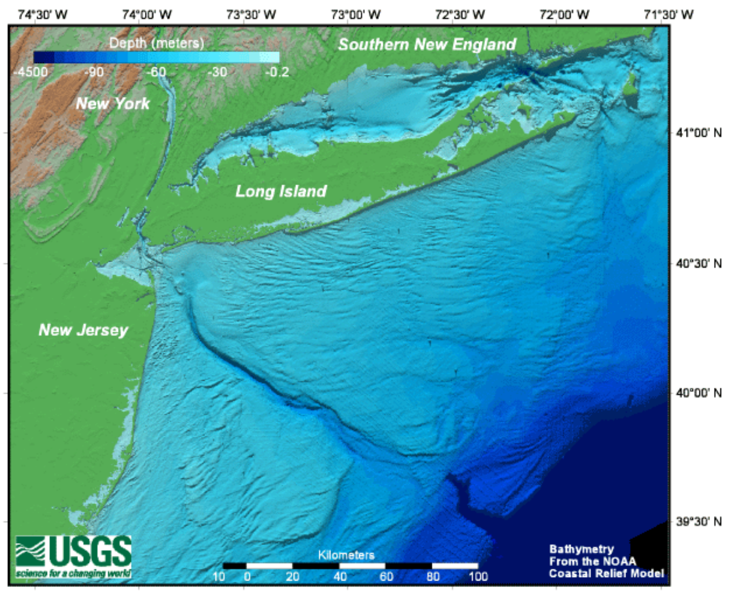
# *Main references*




G.T Csanady: Circulation in the coastal ocean.
Chapter 4. The subtle effects of topography
Section 4.1

# Coastal Bathymetry

Constant depth (flat bottom) conditions, considered so far, are clearly an oversimplification
All natural basins have a complex depth distribution essentially characterised by a depth
Value gradually reducing to zero at the Shore.



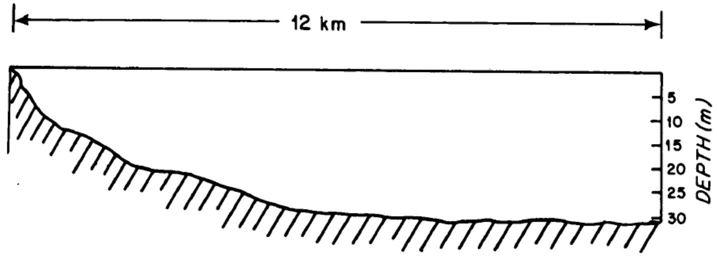




Fig. 4.2. Typical depth distribution close to an open coast: bottom profile off the south coast of Long Island.

# Coastal Bathymetry

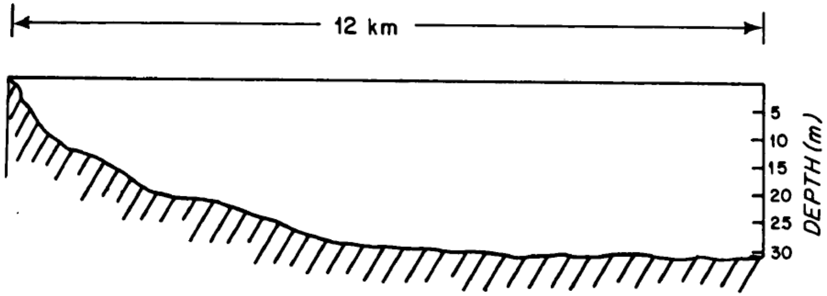
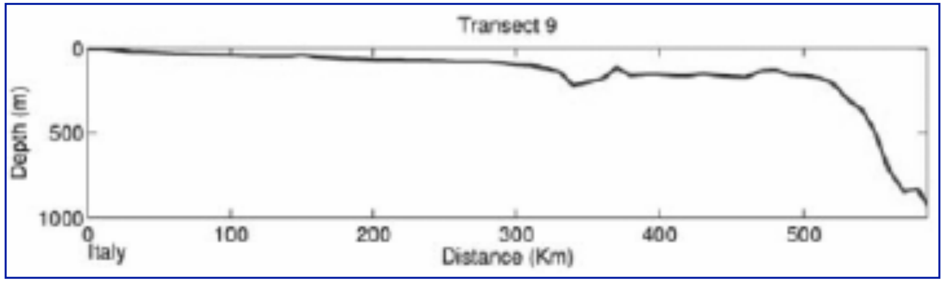



Fig. 4.2. Typical depth distribution close to an open coast: bottom profile off the south coast of Long Island.




The bottom flatness of a typical coastal ocean basin, make the constant depth assumption a valid one in most cases (except very close to the shore, where relative changes in depth are rapid (change in depth with respect to the local depth)
BUT:
There are modes of motion occurring ONLY in a basin with variable depth.

The role of topography in constraining coastal circulation is faced first considering an Homogeneous ocean and then extending the analysis to stratified conditions.

# Wind setup over variable depth

Topographic effect are immediately and intuitively related to the fact that that the wind force distributed over a shallow layer, produces larger average accelerations than the same force acting over deep water.

The wind set up solution (see previous lessons):

$$\frac{\partial \eta}{\partial y} = \frac{u_*^2}{gH} = \frac{u_*^2}{c^2}$$

Indicates that the surface slope is inversely proportional to the (constant) depth

The immediate questions is if there is an analogous solution for  a variable depth basin.

We start by neglecting the bottom stress effects and also (initially) the Coriolis force (assumptions that might be reasonable for small basins and over suitably "short" period of time.

# Wind setup over variable depth

Consider:
A closed basin with arbitrary shape and depth distribution:

$$H=H(x,y)$$

Forced by an (arbitrarily distributed in space) wind stress constant in time.
The linearised transport equations (**neglecting Coriolis force and bottom stress**) are:

$$\frac{\partial U}{\partial t}=-gH\frac{\partial \eta}{\partial x}+\frac{\tau_w^{(x)}}{\rho_0}$$

$$\frac{\partial V}{\partial t}=-gH\frac{\partial \eta}{\partial y}+\frac{\tau_w^{(y)}}{\rho_0}$$

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=-\frac{\partial \eta}{\partial t}$$

# Wind setup over variable depth

$$\frac{\partial U}{\partial t} = -gH\frac{\partial \eta}{\partial x} + \frac{\tau_w^{(x)}}{\rho_0}$$

$$\frac{\partial V}{\partial t} = -gH\frac{\partial \eta}{\partial y} + \frac{\tau_w^{(y)}}{\rho_0}$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = -\frac{\partial \eta}{\partial t}$$

We start by seeking a time independent distribution of $\eta(x,y)$.

Since the forcing is constant in time and $U$ and $V$ are linear in time, we can therefore pose:

$U = At$ and $V = Bt$

Where $A(x, y)$ and $B(x,y)$ are depth integrated accelerations

# Wind setup over variable depth

$$\frac{\partial U}{\partial t} = -gH\frac{\partial \eta}{\partial x} + \frac{\tau_w^{(x)}}{\rho_0}$$

$$\frac{\partial V}{\partial t} = -gH\frac{\partial \eta}{\partial y} + \frac{\tau_w^{(y)}}{\rho_0}$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = -\frac{\partial \eta}{\partial t}$$

The transport equations can then be written as:

$$A = -gH\frac{\partial \eta}{\partial x} + \frac{\tau_w^{(x)}}{\rho_0}$$

$$B = -gH\frac{\partial \eta}{\partial y} + \frac{\tau_w^{(y)}}{\rho_0}$$

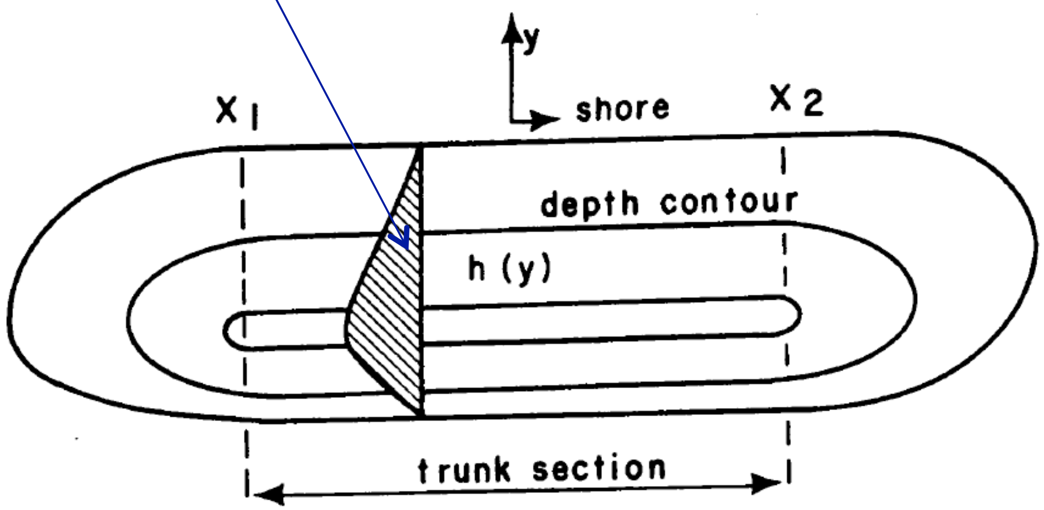
$$\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} = 0$$

Boundary conditions are as usual that normal transport at the shores vanishes.
The solution gives a time independent sea level distribution and a non trivial transport distribution increasing linearly with time.

# Wind setup over variable depth

Consider:
A basin "long and narrow" with parallel depth contours over large part of the basin.

Cross-section

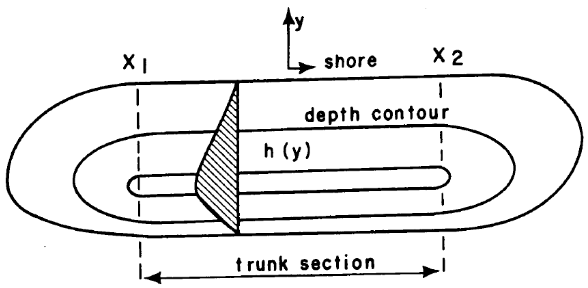



X axis is assumed to be along the length of the basin and the wind stress is acting along such axis.

$$\frac{\tau_w^{(x)}}{\rho_0} = u_*^2 \qquad \frac{\tau_w^{(y)}}{\rho_0} = 0$$

# Wind setup over variable depth

Here we seek the transport distribution along a cross-section ( constant $x$ coordinate) Located in the region with isobaths Parallel to the shores.




At any cross section we have $\int_{y_1}^{y_2} A\,dy = 0$

$y_1$ and $y_2$ are the shore coordinates.

In the region $x_1 < x < x_2$ we assume that transport is only parallel to the shore and to isobaths, because the condition:

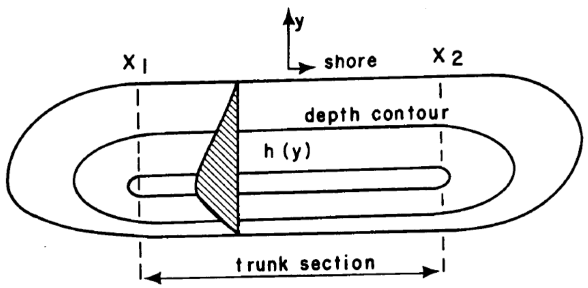
$$\frac{\tau_w^{(y)}}{\rho_0} = 0$$

Yields $\frac{\partial \eta}{\partial y} = 0$ (constant surface elevation along a "$y$" cross-section)

In equation $$B = -gH\frac{\partial \eta}{\partial y} + \frac{\tau_w^{(y)}}{\rho_0}$$

# Wind setup over variable depth

Integrating $A = -gH\dfrac{\partial \eta}{\partial x} + \dfrac{\tau_w^{(x)}}{\rho_0}$ over the cross section




$(x_1<x<x_2)$

And recalling that $\int_{y_1}^{y_2} A\,dy = 0$ , One obtain:

$$\frac{\partial \eta}{\partial x} = \frac{\tau_w^{(x)}}{\rho_0 g H} = \frac{\tau_w^{(x)}(y_2 - y_1)}{\rho_0 g \int_{y_1}^{y_2} H\,dy} = \frac{\tau_w^{(x)}(y_2 - y_1)}{\rho_0 g S}$$

Where:

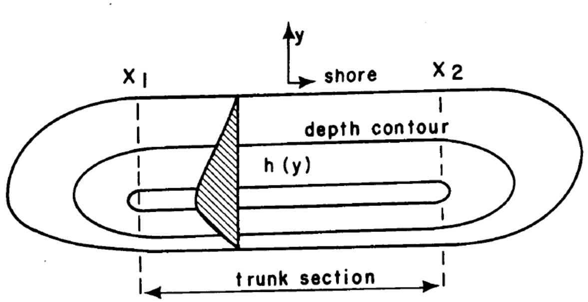
$y_2$-$y_1$ is the width of the cross $S = \int_{y_1}^{y_2} H\,dy$ is the cross sectional area.

We now have $A = \dfrac{U}{t}$ $\dfrac{\partial \eta}{\partial x} = \dfrac{\tau_w^{(x)}(y_2 - y_1)}{\rho_0 g S}$ and we can compute $U$ from

$$A = -gH\frac{\partial \eta}{\partial x} + \frac{\tau_w^{(x)}}{\rho_0}$$

# Wind setup over variable depth

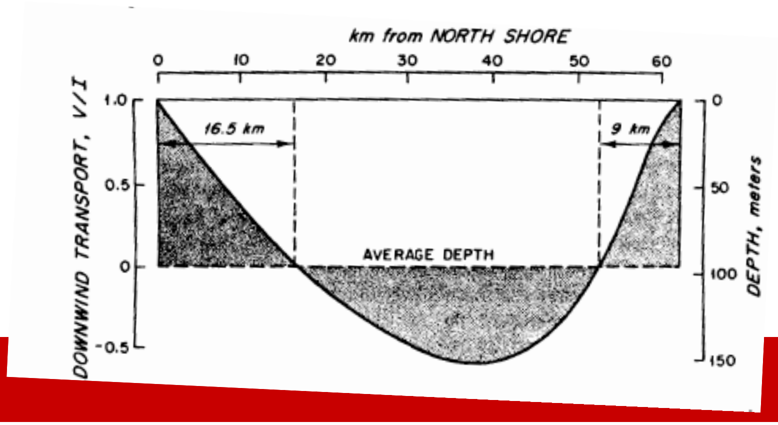
$$A = \frac{U}{t} \qquad \frac{\partial \eta}{\partial x} = \frac{\tau_w^{(x)}(y_2 - y_1)}{\rho_0 g S} \qquad A = -gH\frac{\partial \eta}{\partial x} + \frac{\tau_w^{(x)}}{\rho_0}$$




$$U = \left[-gH\frac{\tau_w^{(x)}(y_2 - y_1)}{\rho g S} + \frac{\tau_w^{(x)}}{\rho_0}\right]t = \frac{\tau_w^{(x)}}{\rho_0}t\left[1 - \frac{H(y_2 - y_1)}{S}\right] \qquad (x_1 < x < x_2)$$

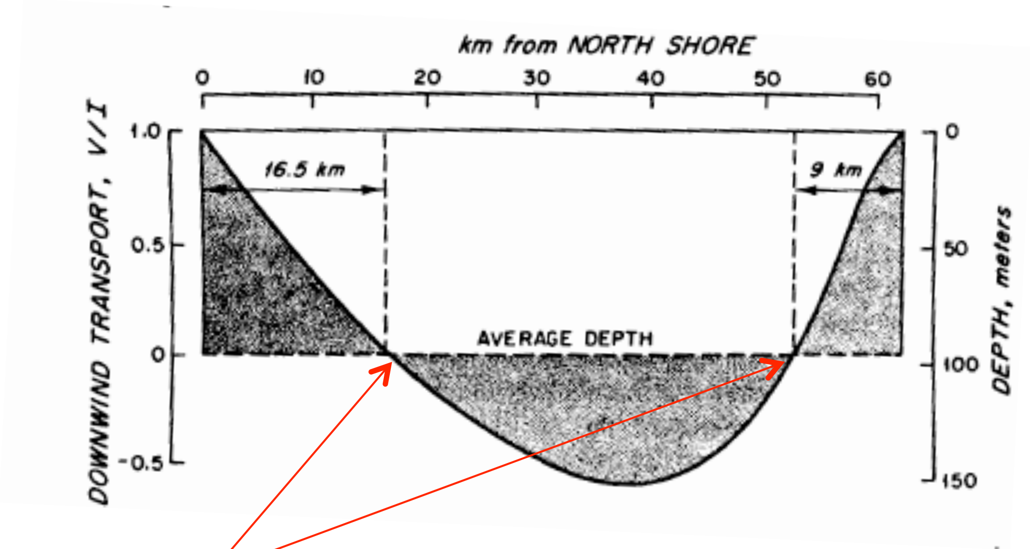
NOTE that: $\frac{S}{(y_2 - y_1)} = \bar{H}$ is the average depth of the section.

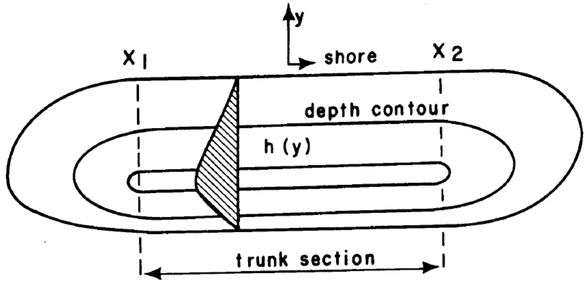
Properties of the solution are discussed using a realistic section




where the transport distribution is described as a rescaled depth.

# Wind setup over variable depth




$$U = \frac{\tau_w^{(x)}}{\rho_0} t \left[ 1 - \frac{H}{\bar{H}} \right]$$

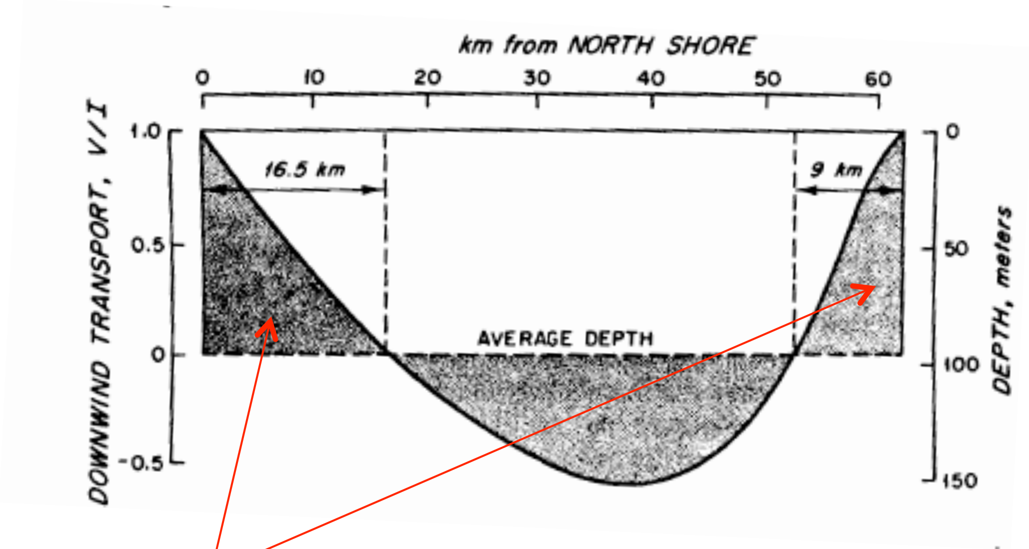
Transport is zero for $H = \bar{H}$ and the elevation gradient

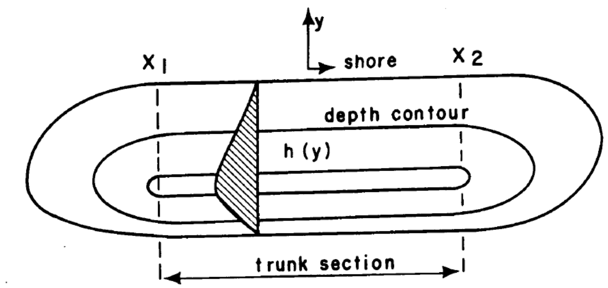
$$\frac{\partial \eta}{\partial x} = \frac{\tau_w^{(x)} \left( y_2 - y_1 \right)}{\rho_0 g S} = \frac{\tau_w^{(x)}}{\rho_0 g \bar{H}} = \frac{\tau_w^{(x)}}{\rho_0 c^2}$$

Is the same as would be produced by wind stress acting over a flat bottom basin with $H = \bar{H}$
Wind stress and pressure gradient are in exact balance and NO transport (down- or up-wind) Is generated.

$$U = \left[ -\bar{H} \frac{\tau_w^{(x)}}{\rho \bar{H}} + \frac{\tau_w^{(x)}}{\rho_0} \right] t = 0$$

# Wind setup over variable depth



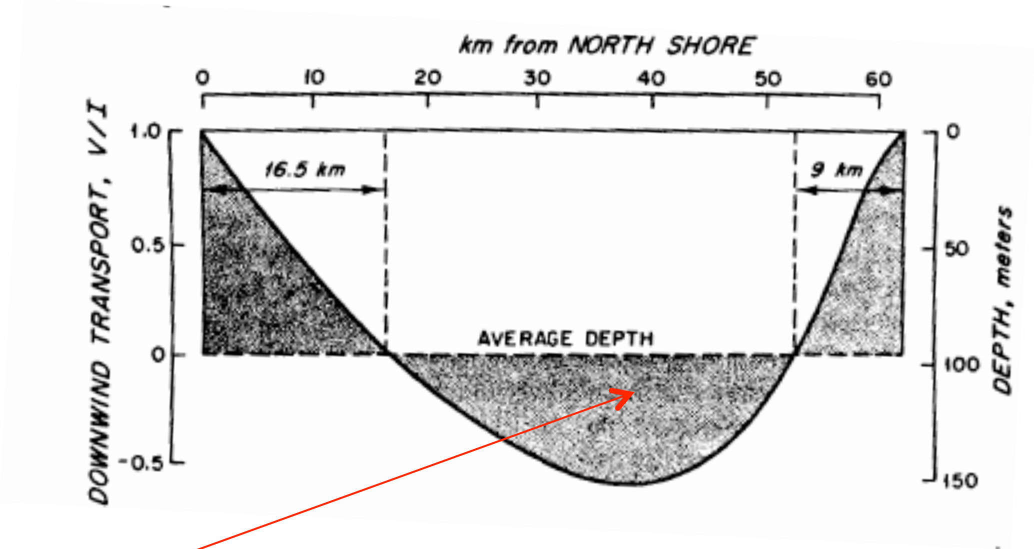

$$U = \frac{\tau_w^{(x)}}{\rho_0} t \left[ 1 - \frac{H}{\bar{H}} \right]$$

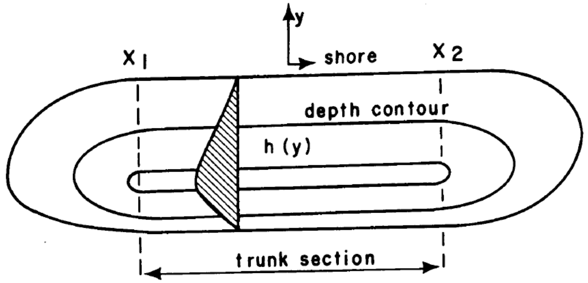
In shallower water where $H < \bar{H}$ the wind stress is greater than the total gravity force $gH \dfrac{\partial \eta}{\partial x}$

Then the wind accelerates water downwind ($U>0$)

$$U = \left[ -H \frac{\tau_w^{(x)}}{\rho \bar{H}} + \frac{\tau_w^{(x)}}{\rho_0} \right] t$$

# Wind setup over variable depth



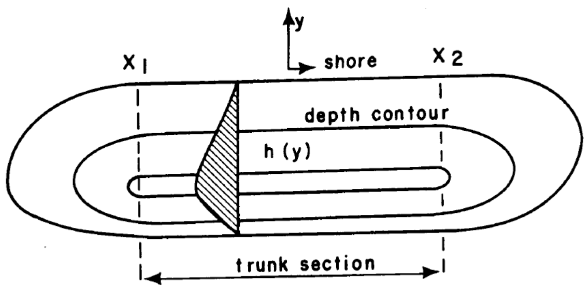

$$U = \frac{\tau_w^{(x)}}{\rho_0} t \left[ 1 - \frac{H}{\bar{H}} \right]$$

In deep water where $H < \bar{H}$ pressure gradient dominates and a return flow $(U<0)$ Is generated.

$$U = \left[ -H \frac{\tau_w^{(x)}}{\rho \bar{H}} + \frac{\tau_w^{(x)}}{\rho_0} \right] t$$

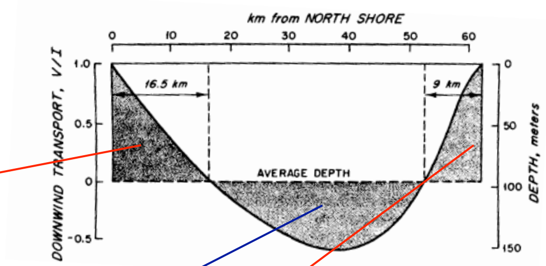
# Wind setup over variable depth

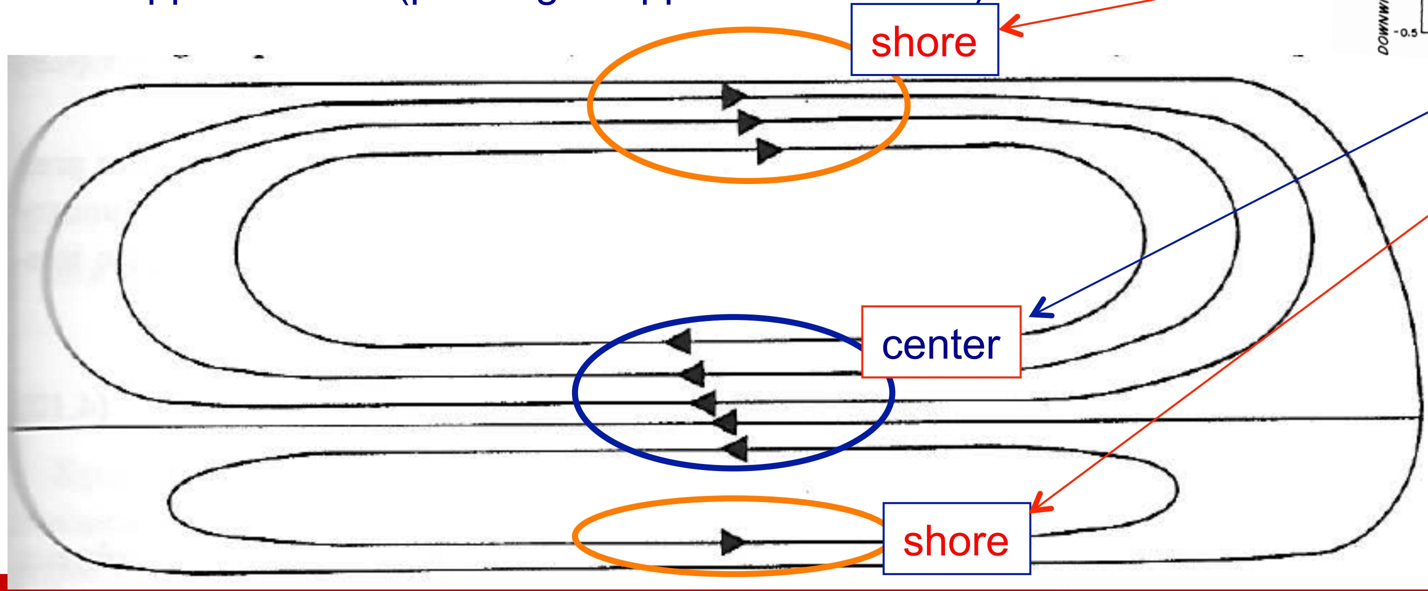
Transport distribution in the remainder of the basin can be Qualitatively described by transport streamlines.




$$U = \frac{\partial \psi}{\partial y} \qquad V = \frac{\partial \psi}{\partial x}$$

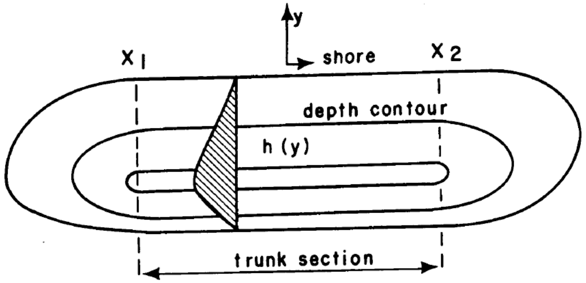
In the region with isobath parallel to the coast, the streamlines correspond to the transport pattern.
This yields densely spaced streamlines at the shores and at the Basin approx center (pointing in opposite directions)

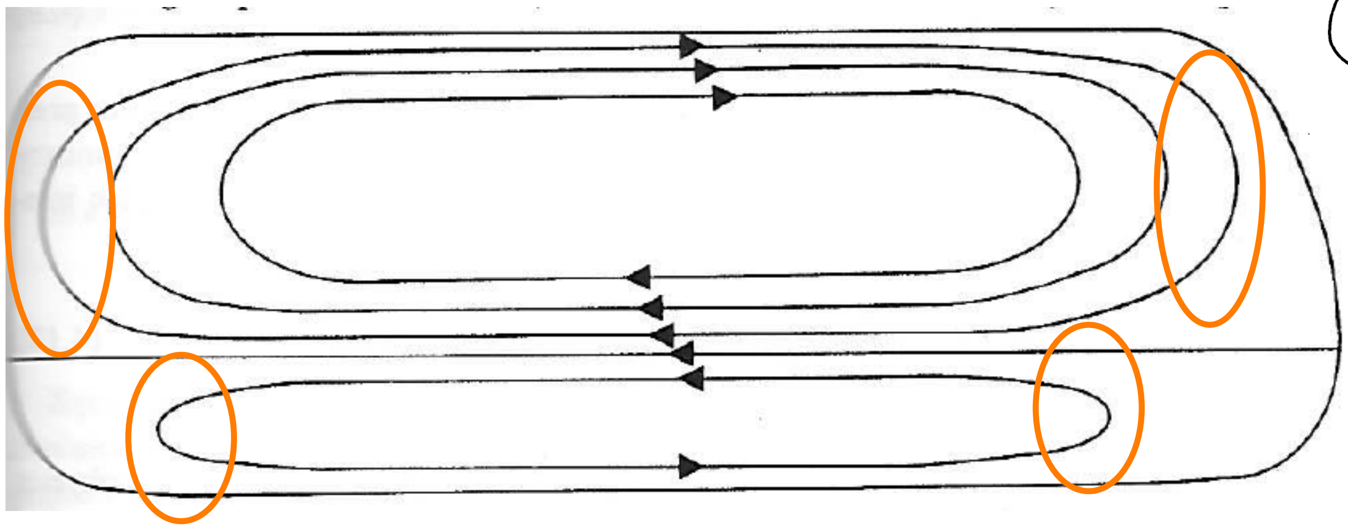


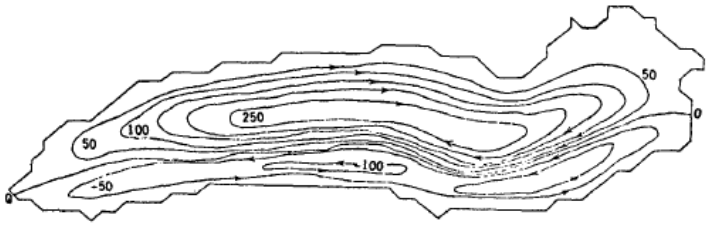
# Wind setup over variable depth

At the "basin end" regions, the streamlines must close



Details of the overall transport fields are depending on the characteristics of the density distribution but the general pattern is that of a "double gyre" . Such gyres are also called "topographic", since are strongly depending on the depth distribution.




Lake Ontario

# *Wind setup over variable depth*

An example: The Gulf of Riga (semi-enclosed basin in the Baltic Sea)


Linear model

1d

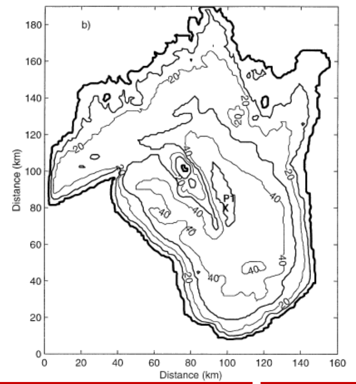
10d




Non Linear model

1d

10d

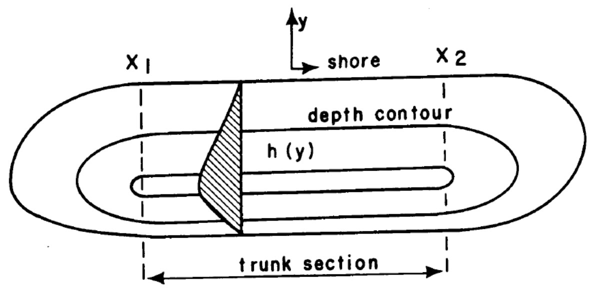




Bathymetry

# Wind setup over variable depth

**Moreover:**

$$U = \frac{\tau_w^{(x)}}{\rho_0} t \left[ 1 - \frac{H}{\overline{\overline{H}}} \right]$$



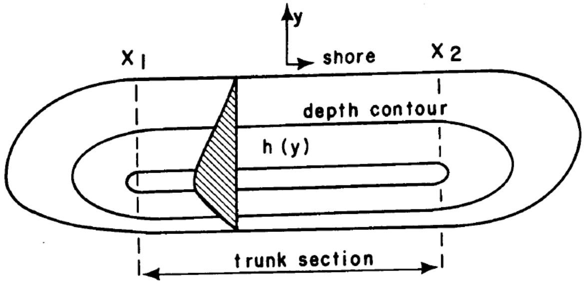

Indicates that the downwind transport is proportional to

$$\frac{\tau_w^{(x)}}{\rho_0} t$$

The windstress impulse.

Nearshore, where $H << \overline{H}$ , the transport is almost equal to the wind stress impulse, as already seen for the along shore wind applied to an homogeneous ocean.
In the shallow depth zone the total pressure force is is proportional to the local depth and it is then almost negligible relative to windstress

# Wind setup over variable depth




The calculated velocity distribution is characterised by:

- Strong coastal currents
- Relatively weak return flow

A wind stress of 0.1 Pa blowing for 10 hr produces an impulse of 3.6 $m^2\ s^{-1}$.

In a basin with $\bar{H} = 100m$ water shallower of 10 m :

- hardly "feel" the pressure gradient force
- wind impulse distributed over a 10 m depth produces a average current of 36 cm s-1.

by contrast:

- Average return flow (upwind leg of topographic gyres)velocities are one order of magnitude less because the larger depth over which the impulse is distributed

CONCLUSION:

- The wind driven coastal current is a very distinct and identifiable feature
- On the contrary many others physical factors can produce a velocity of about 3 cm $s^{-1}$, so the return flow can be "lost' in a very noisy background.